Exact density profiles for the fully asymmetric exclusion process with discrete-time dynamics on semi-infinite chains

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Exact density profiles in the steady state of the one-dimensional fully asymmetric simple-exclusion process on a semi-infinite chain are obtained in the case of forward-ordered sequential dynamics by taking the thermodynamic limit in our recent exact results for a finite chain with open boundaries. The corresponding results for sublattice-parallel dynamics follow from the relationship obtained by Rajewsky and Schreckenberg [Physica A **245**, 139 (1997)], and for parallel dynamics from the mapping found by Evans, Rajewsky, and Speer [J. Stat. Phys. **95**, 45 (1999)]. Our analytical expressions involve Laplace-type integrals, rather than complicated combinatorial expressions, which makes them convenient for taking the limit of a semi-infinite chain, and for deriving the asymptotic behavior of the density profiles at large distances from its end. By comparing the asymptotic results appropriate for parallel update with those published in the above cited paper by Evans, Rajewsky, and Speer, we find complete agreement except in two cases, in which we correct technical errors in the final results given there.

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I. INTRODUCTION

We consider the current and density profiles in the steady state of the fully asymmetric simple-exclusion process (FASEP) with open boundaries and different discrete-time updates, namely, ordered sequential, sublattice-parallel, and parallel. We remind the reader that the model describes a system of particles on a chain, hopping with probability p only to empty nearest-neighbor sites to the right. Each of the L sites of the chain can be either empty or occupied by exactly one particle. Open boundary conditions mean that at each time step (update of the whole chain) a particle is injected with probability α at the left end of the chain ($i=1$), and removed with probability β at the right end ($i=L$). The definition of the model includes the choice of the stochastic dynamics, i.e., the update scheme which specifies the order in which the local hopping, injection, and particle removal are implemented. The case of random-sequential update is considered as a lattice automaton realization of the corresponding continuous-time process. In the general case it was solved first by using the recursion relation method [1,2], and then by means of the elegant matrix-product ansatz (MPA) [3]. As was proved later [4], the MPA is actually not an ansatz, but an exact representation of the stationary state of any one-dimensional system with random-sequential dynamics involving nearest-neighbor hoppings and single-site boundary terms. The method of the MPA was next successfully applied for solving the following basic cases of true discrete-time dynamics: forward-ordered sequential ($\rightarrow$), backward-ordered sequential ($\leftarrow$) [5,6], and sublattice-parallel (S-$\|$) [7,8], which turned out to be closely related [9]. Thus, the current has the same value in all these cases, $J_L^{\rightarrow}=J_L^{\leftarrow}=J_L^{S\text{-}\|}$; the local densities at site $i\in\{1,\ldots,L\}$ for the FASEP with forward-, $\rho_L^{\rightarrow}(i)$, and backward-, $\rho_L^{\leftarrow}(i)$, ordered sequential updates are simply related to each other, $\rho_L^{\rightarrow}(i)=\rho_L^{\leftarrow}(i)-J_L^{\rightarrow}$, and to the local density $\rho_L^{S\text{-}\|}(i)$ for the sublattice-parallel update [8],

$$\rho_L^{S\text{-}\|}(i)=\begin{cases}\rho_L^{\rightarrow}(i), & i \ \text{odd}\\ \rho_L^{\leftarrow}(i), & i \ \text{even.}\end{cases} \tag{1}$$

In the cases of random-sequential, ordered sequential, and sublattice-parallel updates, the matrix-product representation involves infinite-dimensional matrices that satisfy a particular quadratic algebra. Finally, the most difficult case of fully parallel dynamics and open boundaries was solved by Evans, Rajewsky, and Speer [10], by using site-oriented MPA with matrices satisfying a quartic algebra. Moreover, these authors showed that the current and local densities for the model with parallel update can be simply mapped onto those for the previously solved discrete-time updates. An independent, bond-oriented MPA solution for the stationary FASEP problem with parallel dynamics was found by de Gier and Nienhuis [11]. In the case of general values of the hopping probability p, they presented explicit expressions for the current and the discrete slope $t_L(i)=\rho_L(i+1)-\rho_L(i)$ of the density profile in all qualitatively different domains of the parameter space.

In our recent paper [12] we derived, independently of [10,11] and by using a different method, exact expressions for the steady state current $J_L^{\rightarrow}$ and the local density $\rho_L^{\rightarrow}(i)$, $i\in\{1,\ldots,L\}$, for the FASEP on a finite chain with forward-ordered sequential dynamics and open boundaries. These expressions involve integrals that at large L and large distance from the chain ends are of the Laplace type; hence they are convenient for asymptotic analysis. We point out that from the simple mapping [10] of the above quantities onto the current $J_L^{\|}$ and local density $\rho_L^{\|}(i)$ for the model with parallel update,

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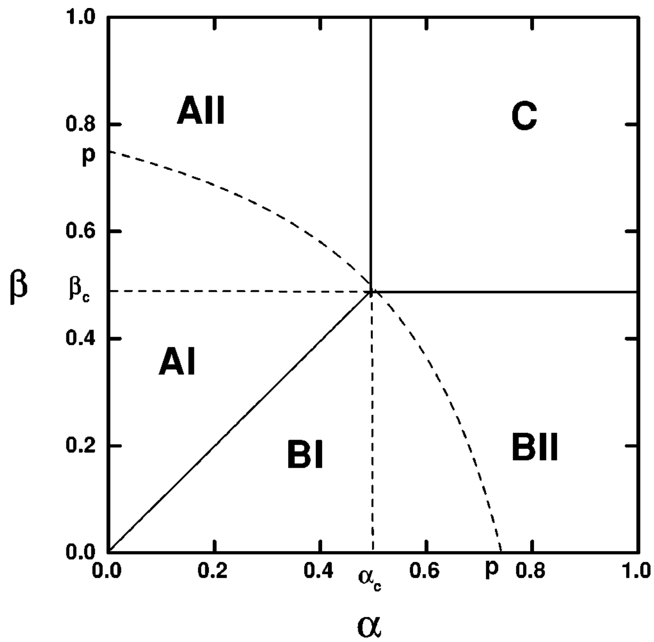

FIG. 1. The phase diagram in the plane of the injection and removal probabilities α and β (see the text) for hopping probability $p=0.75$. The maximum-current phase occupies region C. Region $A=A\mathrm{I}\cup A\mathrm{II}$ corresponds to the low-density phase, and region $B=B\mathrm{I}\cup B\mathrm{II}$ to the high-density phase. Subregions AI (BI) and AII (BII) are distinguished by the different analytic forms of the density profile. The boundary between them, $\beta=\beta_c$, $0\leqslant\alpha\leqslant\alpha_c$ ($\alpha=\alpha_c, 0\leqslant\beta\leqslant\beta_c$), is shown by the dashed segment of a straight line. The solid line $\alpha=\beta$ between subregions AI and BI is the coexistence line of the low- and high-density phases. The curved dashed line is the mean-field line $(1-\alpha)(1-\beta)=1-p$.

$$J_L^{\parallel}=\frac{J_L^{\rightarrow}}{1+J_L^{\rightarrow}},\quad \rho_L^{\parallel}(i)=\frac{\rho_L^{\rightarrow}(i)+J_L^{\rightarrow}}{1+J_L^{\rightarrow}},\tag{2}$$

exact representations for $J_L^{\parallel}$ and $\rho_L^{\parallel}(i)$ follow that do not contain complicated combinatorial expressions like those obtained by Evans, Rajewsky, and Speer.

We shall present here exact expressions for the local density profiles of the model on a semi-infinite chain with either left-hand (l) end point,

$$\rho_{\infty,\mathrm{l}}^{\rightarrow}(i|\alpha,\beta)=\lim_{L\to\infty}\rho_L^{\rightarrow}(i|\alpha,\beta),\tag{3}$$

or right-hand (r) end point,

$$\rho_{\infty,\mathrm{r}}^{\rightarrow}(j|\alpha,\beta)=\lim_{L\to\infty}\rho_L^{\rightarrow}(L-j+1|\alpha,\beta).\tag{4}$$

In the former case the limit $L\to\infty$ is taken at fixed $i=1,2,\ldots$, which labels the sites of the chain from its left end to the right. In the latter case the limit $L\to\infty$ is taken at fixed $j=1,2,\ldots$, which labels the sites of the chain from its right end to the left. Note that for fixed hopping probability p, and any point (α,β) on the phase diagram (see Fig. 1), the right-hand profile $\rho_{\infty,\mathrm{r}}^{\rightarrow}(i|\alpha,\beta)$ is closely related to the left-hand profile $\rho_{\infty,\mathrm{l}}^{\rightarrow}(i|\beta,\alpha)$ for a point that is symmetrically positioned with respect to the diagonal $\alpha=\beta$. Indeed, by taking the $L\to\infty$ limit in the symmetry relation for a finite chain [9], $\rho_L^{\rightarrow}(i|\alpha,\beta)=1-J_L^{\rightarrow}-\rho_L^{\rightarrow}(L-i+1|\beta,\alpha)$, one obtains that

$$\rho_{\infty,\mathrm{l}}^{\rightarrow}(i|\alpha,\beta)=1-J_{\infty}^{\rightarrow}-\rho_{\infty,\mathrm{r}}^{\rightarrow}(i|\beta,\alpha).\tag{5}$$

Therefore, it suffices to derive explicit expressions for the local densities $\rho_{\infty,\mathrm{l}}^{\rightarrow}(i|\alpha,\beta)$ and $\rho_{\infty,\mathrm{r}}^{\rightarrow}(j|\alpha,\beta)$ for, say, $\alpha\leqslant\beta$, i.e., in the maximum-current and low-density phases. Moreover, by applying the $L\to\infty$ limit form of the mapping (2) with i fixed at a finite distance from one of the chain ends (e),

$$J_{\infty}^{\parallel}=\frac{J_{\infty}^{\rightarrow}}{1+J_{\infty}^{\rightarrow}},\quad \rho_{\infty,\mathrm{e}}^{\parallel}(i)=\frac{\rho_{\infty,\mathrm{e}}^{\rightarrow}(i)+J_{\infty}^{\rightarrow}}{1+J_{\infty}^{\rightarrow}}\ (\mathrm{e=l,r}),\tag{6}$$

to the results for $\rho_{\infty,\mathrm{e}}^{\rightarrow}(i)$, one obtains the corresponding expressions for $\rho_{\infty,\mathrm{e}}^{\parallel}(i)$, $i=1,2,\ldots$, e=l,r. We emphasize that the resulting expressions for the density profiles on semi-infinite chains are much simpler than those on finite chains, and they again involve integrals that are of the Laplace type for large distances from the chain end. Hence, their asymptotic behavior is readily deduced. In order to make precise comparison with the asymptotic expansions for the right-hand local density profiles obtained in [10], one has to take into account that there, before taking the limit $L\to\infty$, the sites are labeled from the right end of the chain to the left by $n=L-i=0,1,2,\ldots$, rather than by $j=L-i+1=1,2,\ldots$, as in our definition (4). By comparing the large j asymptotic behavior of $\rho_{\infty,\mathrm{r}}^{\parallel}(j)$, obtained via the mapping (6) with the corresponding results of Evans, Rajewsky, and Speer [10] for ρ_n with $n=j-1$, we find complete agreement in all but two cases, in which we correct technical errors in the final asymptotic expressions (9.22) and (9.28) given there. The expressions for the slope of the density profile derived in [11] for general values of the hopping probability p actually represent the leading-order asymptotic forms of $t_L(i)$ that follow from our expressions for the local density $\rho_L(i)$ [12] in the semi-infinite chain limit $L\to\infty$ and in the leading-order expansion in finite but large $L-i=n\gg 1$; see the results and comments below. For recent general reviews on the MPA, ASEP, and related systems we refer the reader to [13–15].

Concerning the notation, we make the following remarks. In contrast to our previous work [12], here we shall adhere to the conventional labeling of the different regions in the phase diagram (see Fig. 1). The mean field line shown there is given by the equation $(1-\alpha)(1-\beta)=1-p$, and the two other special lines are defined by

$$\alpha=\alpha_c\equiv 1-\sqrt{1-p},\quad \beta=\beta_c\equiv 1-\sqrt{1-p}.\tag{7}$$

The exact finite-chain results obtained in [12] are conveniently expressed in terms of the parameters

$$d=\sqrt{1-p},\quad a=d+d^{-1},\quad \xi=\frac{p-\alpha}{\alpha d},\quad \eta=\frac{p-\beta}{\beta d},\tag{8}$$

which will be used here too.

II. LOCAL DENSITY IN THE MAXIMUM-CURRENT PHASE

The maximum-current phase occupies the region $\alpha_c<\alpha\leqslant 1$ and $\beta_c<\beta\leqslant 1$; see region C in Fig. 1. In terms of the variables (8) the above inequalities read $-d\leqslant\xi<1$ and $-d\leqslant\eta<1$.

The final exact result obtained in [12] for the local particle density in the maximum-current phase is

$$\rho_L(i)=\frac{1}{2}(1-J_L)+\frac{d}{2pZ_L}[F_L(i)-(\xi-\eta)Z_{i-1}Z_{L-i}]. \tag{9}$$

Here $Z_n=Z_n(\xi,\eta)$ has the representation ($\xi\neq\eta$)

$$Z_n(\xi,\eta)=\left(\frac{d}{p}\right)^n\left[\frac{\xi}{\xi-\eta}I_n(\xi)+\frac{\eta}{\eta-\xi}I_n(\eta)\right], \tag{10}$$

which involves the integral

$$I_n(\xi)=\frac{2}{\pi}\int_0^\pi d\phi\,\frac{(a+2\cos\phi)^n\sin^2\phi}{1-2\xi\cos\phi+\xi^2}. \tag{11}$$

The expression for $Z_n(\xi,\xi)$ follows by taking the limit $\eta\to\xi$ in Eq. (10). The current $J_L(\xi,\eta)$ is given by

$$J_L(\xi,\eta)=Z_{L-1}(\xi,\eta)/Z_L(\xi,\eta). \tag{12}$$

The term $F_L(i)=F_L(i;\xi,\eta)$ in Eq. (9) is an antisymmetric (with respect to the center of the chain) function of the integer coordinate i, $F_L(i;\xi,\eta)=-F_L(L-i+1;\xi,\eta)$, defined for $1\leqslant i\leqslant[L/2]$ ($[x]$ denotes the integer part of x) by the equation

$$F_L(i;\xi,\eta)=\left(\frac{d}{p}\right)^{L-1}(1-\xi\eta)\sum_{n=0}^{L-2i}I_{L-i-n-1}(\xi)I_{i+n-1}(\eta). \tag{13}$$

In order to take the limit $L\to\infty$ in Eq. (9), we make use of the large-n asymptotic form of the Laplace-type integral (11):

$$I_n(\xi)=\frac{(a+2)^{n+3/2}}{2\sqrt{\pi}(1-\xi)^2}n^{-3/2}[1+O(n^{-1})]\quad(\xi\neq 1)$$

$$I_n(1)=\frac{(a+2)^{n+1/2}}{\sqrt{\pi}}n^{-1/2}[1+O(n^{-1})]. \tag{14}$$

By substituting the above expansion in Eq. (10), we obtain that in the maximum-current (MC) phase the leading asymptotic form of $Z_n(\xi,\eta)$ is ($\xi\neq 1,\eta\neq 1$)

$$Z_n^{\mathrm{MC}}(\xi,\eta)=\frac{1-\xi\eta}{2\sqrt{\pi}(1-\xi)^2(1-\eta)^2}\left(\frac{d}{p}\right)^n\times\frac{(a+2)^{n+3/2}}{n^{3/2}}[1+O(n^{-1})]. \tag{15}$$

Hence, the asymptotic form of the current reads

$$J_L^{\mathrm{MC}}=\frac{1-\sqrt{1-p}}{1+\sqrt{1-p}}[1+O(L^{-1})], \tag{16}$$

independently of the parameters α and β. With the aid of Eq. (15), at fixed i we obtain

$$\lim_{L\to\infty}\frac{d}{2p}\frac{Z_{i-1}Z_{L-i}}{Z_L}=\frac{\xi I_{i-1}(\xi)-\eta I_{i-1}(\eta)}{2(\xi-\eta)(a+2)^i}. \tag{17}$$

Next, we split the sum over n in Eq. (13) into two parts, from 0 to $[L/2]-i$ and from $[L/2]-i+1$ to $L-2i$, and replace the cofactor in the product $I_{L-i-n-1}(\xi)I_{i+n-1}(\eta)$ which has a large subscript as $L\to\infty$ by its asymptotic form (14). Thus, we prove the limit

$$\lim_{L\to\infty}\frac{d}{2p}\frac{F_L(i;\xi,\eta)}{Z_L(\xi,\eta)}=\frac{1}{2}\sum_{n=0}^{\infty}\left[\frac{(1-\xi)^2I_{i+n-1}(\xi)}{(a+2)^{i+n+1}}+\frac{(1-\eta)^2I_{i+n-1}(\eta)}{(a+2)^{i+n+1}}\right]. \tag{18}$$

Finally, by using the identity

$$\frac{1}{2}\sum_{n=0}^{\infty}\frac{(1-\xi)^2I_{i+n-1}(\xi)}{(a+2)^{i+n+1}}=\frac{G_{i-1}-\xi I_{i-1}(\xi)}{(a+2)^i}, \tag{19}$$

where

$$G_n=\frac{1}{\pi}\int_0^\pi d\phi(a+2\cos\phi)^n(1+\cos\phi), \tag{20}$$

and collecting the above results, we obtain an exact expression for the local density profile of a semi-infinite chain with left-hand end point in the form

$$\rho_{\infty,1}^{\rightarrow}(i)=\frac{d}{1+d}+\frac{1}{(a+2)^i}[G_{i-1}-\xi I_{i-1}(\xi)]. \tag{21}$$

The approach of the left-hand profile for finite chains, obtained by computer simulations, to the limit (21) as the chain size increases is illustrated in Fig. 2.

The asymptotic behavior for $i\gg 1$ readily follows from the expansions (14) and

$$G_{i-1}=\frac{(a+2)^{i-1/2}}{\sqrt{\pi}}\left[i^{-1/2}-\frac{a}{16}i^{-3/2}+O(i^{-5/2})\right]. \tag{22}$$

Thus we obtain

$$\rho_{\infty,1}^{\rightarrow}(i)=\frac{d}{1+d}+\frac{d^{1/2}}{\sqrt{\pi}(1+d)}i^{-1/2}-\frac{1}{\sqrt{\pi}(1+d)}\left[\frac{2-p}{16d^{1/2}}+\frac{\alpha d^{1/2}(p-\alpha)}{2(\alpha-\alpha_c)^2}\right]i^{-3/2}+O(i^{-5/2}). \tag{23}$$

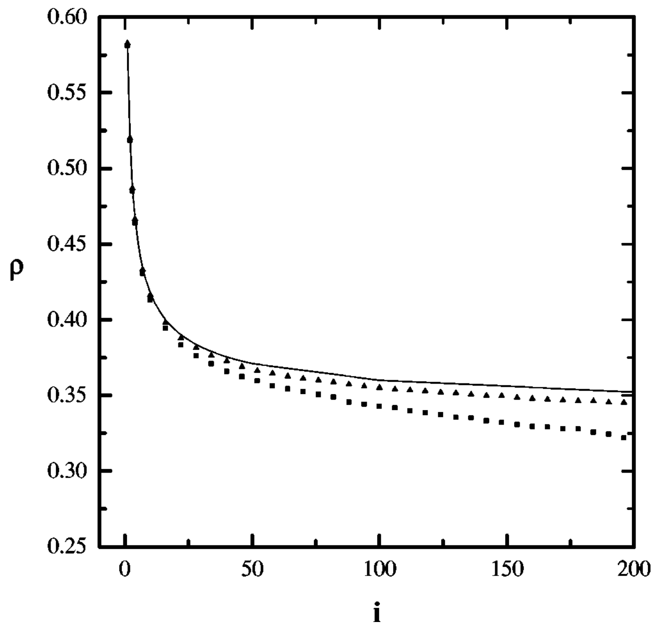

FIG. 2. The approach of the left-hand local number-density profile ρ versus the site number i in the maximum-current phase ($\alpha=\beta=0.8$, $p=0.75$), obtained by computer simulations for finite chains of length L [solid squares, $L=300$, solid triangles, $L=1000$ (data averaged over 300 runs of 2^{16} Monte Carlo steps per site)], to the limit given by Eq. (21) (solid line).

Hence, by using the symmetry relation (5) and the mapping (6) onto the case of parallel update, after taking into account the correspondence $n=i-1$ with the expansion variable used in [10], we obtain

$$\rho_n \equiv \rho^{\parallel}_{\infty,\mathrm{r}}(n+1)=\frac{1}{2}-\frac{d^{1/2}}{2\sqrt{\pi}}n^{-1/2}+\left[\frac{2-p+8d}{32\sqrt{\pi}d^{1/2}}+\frac{\beta d^{1/2}(p-\beta)}{4\sqrt{\pi}(\beta-\beta_c)^2}\right]n^{-3/2}+O(n^{-5/2}). \tag{24}$$

This result coincides with Eq. (9.22) of Ref. [10] except for the sign of the $O(n^{-1/2})$ term. The first two terms in the right-hand side of Eq. (24) generate Eq. (87) in [11], provided the slope of the density profile is considered there near the right-hand boundary of a semi-infinite chain, i.e., when $L\to\infty$, $i/L\to 1$.

III. LOCAL DENSITY IN THE LOW-DENSITY PHASE

The low-density phase occupies the region $\alpha<\beta$ and $\alpha<\alpha_c$; see region $A=A\mathrm{I}\cup A\mathrm{II}$ in Fig. 1. In the case of a finite chain of L sites, up to corrections exponentially small in L the current in the low-density (LD) phase is [12]

$$J_L^{\mathrm{LD}}(\xi,\eta)\simeq\frac{p}{d(a+\xi+\xi^{-1})}=\frac{\alpha(p-\alpha)}{p(1-\alpha)}. \tag{25}$$

With respect to the asymptotic form of the density profile near the right-hand end of the chain, one distinguishes two subregions, AI and AII.

Subregion AI. $\alpha<\alpha_c$ and $\alpha<\beta<\beta_c$ $(\xi>\eta>1)$. As shown in [12], up to terms that are uniformly in $i=1,\dots,L$ exponentially small as $L\to\infty$, the local density is

$$\rho_L(i;\xi,\eta)\simeq\frac{\alpha(1-p)}{p(1-\alpha)}+\frac{\eta-\eta^{-1}}{a+\xi+\xi^{-1}}\left(\frac{a+\eta+\eta^{-1}}{a+\xi+\xi^{-1}}\right)^{L-i}-\frac{\xi I_{L-i}(\xi)-\eta I_{L-i}(\eta)}{(a+\xi+\xi^{-1})^{L-i+1}}. \tag{26}$$

Hence, in the limit $L\to\infty$ at fixed i, we obtain that $\rho^{\rightarrow}_{\infty,\mathrm{l}}(i)$ is uniform, equal to the bulk density given by the first term in the right-hand side of Eq. (26). If the limit $L\to\infty$ is taken at fixed $j=L-i+1$, we obtain the exact expression for the nontrivial density profile on a semi-infinite chain with right-hand end point:

$$\rho^{\rightarrow}_{\infty,\mathrm{r}}(j)=\frac{\alpha(1-p)}{p(1-\alpha)}+\frac{\eta-\eta^{-1}}{a+\eta+\eta^{-1}}\left(\frac{a+\eta+\eta^{-1}}{a+\xi+\xi^{-1}}\right)^{j}-\frac{\xi I_{j-1}(\xi)-\eta I_{j-1}(\eta)}{(a+\xi+\xi^{-1})^{j}}. \tag{27}$$

At large distance from the end of the chain, $j\gg 1$, the leading-order asymptotic form of $\rho^{\rightarrow}_{\infty,\mathrm{r}}(j)$ is given by the first two terms in the right-hand side of Eq. (27). Upon mapping on the case of parallel dynamics [see Eq. (6)], and taking into account the labeling correspondence $j=n+1$, our result coincides with Eq. (9.25) of [10]. Thus, the leading-order expression for the slope of the density profile as $n\to\infty$ exactly reproduces Eq. (82) in [11].

Subregion AII. $\alpha<\alpha_c$ and $\beta>\beta_c$ $(\xi>1$ and $\eta<1)$. As shown in [12], up to terms that are uniformly in $i=1,\dots,L$ exponentially small as $L\to\infty$, the local density in this subregion is

$$\rho_L(i;\xi,\eta)\simeq\frac{\alpha(1-p)}{p(1-\alpha)}-\frac{\xi I_{L-i}(\xi)-\eta I_{L-i}(\eta)}{(a+\xi+\xi^{-1})^{L-i+1}}. \tag{28}$$

Hence, in the limit $L\to\infty$ at fixed i, we obtain that $\rho^{\rightarrow}_{\infty,\mathrm{l}}(i)$ is again uniform, equal to the bulk density. When the limit $L\to\infty$ is taken at fixed $j=L-i+1$, we obtain the exact expression for the nontrivial density profile on a semi-infinite chain with right-hand end point:

$$\rho^{\rightarrow}_{\infty,\mathrm{r}}(j)=\frac{\alpha(1-p)}{p(1-\alpha)}-\frac{\xi I_{j-1}(\xi)-\eta I_{j-1}(\eta)}{(a+\xi+\xi^{-1})^{j}}. \tag{29}$$

At large distance from the right-hand end point, $j\gg 1$, the leading-order asymptotic form of $\rho^{\rightarrow}_{\infty,\mathrm{r}}(j)$ in subregion AII is

$$\rho^{\rightarrow}_{\infty,\mathrm{r}}(j)=\frac{\alpha(1-p)}{p(1-\alpha)}-\frac{\sqrt{a+2}}{2\sqrt{\pi}}\left[\frac{\xi}{(1-\xi)^2}-\frac{\eta}{(1-\eta)^2}\right]\times\left(\frac{a+2}{a+\xi+\xi^{-1}}\right)^{j}j^{-3/2}+O(j^{-5/2}). \tag{30}$$

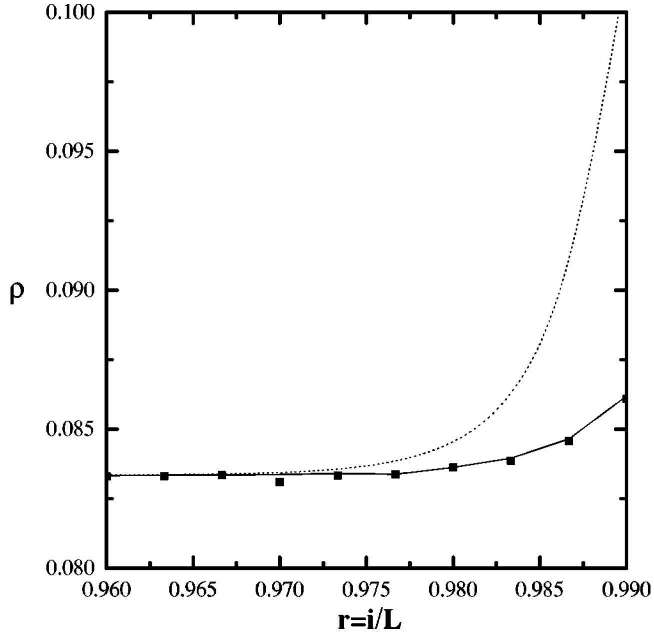

FIG. 3. The behavior of the right-hand number-density profile ρ versus the scaled distance $r=i/L$ in subregion AII ($\alpha=0.2$, $\beta=0.6$, $p=0.75$): the solid squares present the results of computer simulations for a finite chain of $L=300$ sites, the solid line corresponds to Eq. (29) for $L=300$, and the dotted line to the leading-order asymptotic form of the profile for a semi-infinite chain given by Eq. (30).

A comparison of the right-hand density profile for a finite chain [Eq. (28)] and its leading-order asymptotic form on a semi-infinite chain [Eq. (30)] with the result of computer simulations for $L=300$ is given in Fig. 3.

Upon mapping on the case of parallel dynamics [see Eq. (6)], expressing the result in terms of the original parameters α and β, and taking into account the labeling correspondence $j=n+1$, we obtain

$$\rho^{\parallel}_{\infty,\mathrm{r}}(n)=\frac{\alpha(1-\alpha)}{p-\alpha^2}-\frac{\alpha(p-\alpha)}{2\sqrt{\pi}(p-\alpha^2)}\frac{d^{1/2}}{1-d}\left[\frac{\alpha(p-\alpha)}{(\alpha-\alpha_c)^2}-\frac{\beta(p-\beta)}{(\beta-\beta_c)^2}\right]\left[\frac{\alpha(p-\alpha)}{(1-\alpha)(1-d)^2}\right]^n n^{-3/2}+O\left(n^{-5/2}\left[\frac{\alpha(p-\alpha)}{(1-\alpha)(1-d)^2}\right]^n\right). \tag{31}$$

This is the correct version of Eq. (9.28) in [10]; the error there occurred on passing to the second equality in Eq. (9.27). For the slope of the density profile the first two terms in the right-hand side of Eq. (31) yield Eq. (84) in [11] as a leading-order expression when $n\gg 1$.

The boundary between subregions AI and AII. $\alpha<\alpha_c$ and $\beta=\beta_c$ ($\xi>1$ and $\eta=1$). The exact expression for the density profile on a semi-infinite chain with right-hand end point is given by Eq. (29) at $\eta=1$. Its large-distance asymptotic form is readily obtained with the aid of expansions (14):

$$\rho^{\rightarrow}_{\infty,\mathrm{r}}(j)=\frac{\alpha(1-p)}{p(1-\alpha)}+\frac{1}{\sqrt{\pi}\sqrt{a+2}}\left(\frac{a+2}{a+\xi+\xi^{-1}}\right)^j j^{-1/2}+O(j^{-3/2}). \tag{32}$$

Upon mapping on the case of parallel dynamics with the aid of Eq. (6), we recover exactly Eq. (9.29) in [10]. Thus, for the leading-order asymptotic form of the slope of the density profile as $n\to\infty$ one obtains Eq. (83) in [11].

IV. DISCUSSION

Summarizing, we have calculated exactly the local density profiles in all the phases of the FASEP on semi-infinite chains with open left-hand (particle injection) or right-hand (particle removal) boundary. The expressions obtained are much simpler than the finite-size ones, and they readily admit large-distance asymptotic analysis.

The following general features of the local density profiles on semi-infinite chains turn out to be common to all the basic updates.

(i) The profile in the maximum-current phase depends on the boundary condition at the finite end point of the chain, and *does not* depend on the boundary condition at infinity. The leading-order approach to the bulk density with the distance i from the chain end is of the order $O(i^{-1/2})$ and does not depend on the boundary conditions at all.

(ii) The local density is uniform in the low-density phase of a chain with left-hand end point, and in the high-density phase of a chain with right-hand end point.

(iii) The local density profiles in the low-density phase of a chain with right-hand end point, and in the high-density phase of a chain with left-hand end point, depend on the boundary conditions *both* at the finite end point and at infinity. This fact is reminiscent of the dependence of equilibrium states with spontaneously broken symmetry on boundary conditions at infinity. However, it is the continuity of the current that here keeps the information about the boundary condition at the chain end that goes to infinity in the limit $L\to\infty$.

(iv) The analytic form of the local density profile on a semi-infinite chain with right-hand (left-hand) end point changes on passing from subregion AI (BI) to subregion AII (BII) within the low-density (high-density) phase [compare Eqs. (27) and (29)]. The leading-order large-distance asymptotic approach to the bulk density is also qualitatively different: in AI (BI) the approach is purely exponential, with inverse correlation length $\lambda^{-1}=|\lambda_\xi^{-1}-\lambda_\eta^{-1}|$, while in AII (BII) the approach is exponential, with power-law prefactor $j^{-3/2}$ and inverse correlation length $\lambda_\xi^{-1}(\lambda_\eta^{-1})$, where

$$\lambda_\xi^{-1}=\ln\left(\frac{a+\xi+\xi^{-1}}{a+2}\right). \tag{33}$$

On the borderline between these two subregions the leading-order asymptotic approach is exponential, with power-law prefactor $j^{-1/2}$ and inverse correlation length given by $\lambda_\xi^{-1}(\lambda_\eta^{-1})$. On crossing the mean-field line in subregion AII (BII), the analytic form of the profile does not change, only its bending near the right-hand (left-hand) end changes from downward (upward) above that line to upward (downward) below it [see Eq. (30)].

(v) The correlation length depends on the type of update only: for all the true discrete-time updates λ_ξ is given by Eq. (33), and for the random-sequential update [see Eq. (78) in [3]]

$$\lambda_\alpha^{-1}=\ln\left(\frac{1}{4\alpha(1-\alpha)}\right). \tag{34}$$

We point out that from the mapping (2) of our exact results for the finite-chain current $J_L^{\rightarrow}$ and local density $\rho_L^{\rightarrow}(i)$ [12] onto the case of parallel update, exact representations for $J_L^{\parallel}$ and $\rho_L^{\parallel}(i)$ follow, which, in contrast to those found in [10], are convenient for asymptotic analysis. Finally, by comparing the asymptotic results so obtained to those published in [10], we have corrected technical errors in two of the expressions given there. The leading-order large-distance expressions that follow for the slope of the density profile near the right-hand end point of a semi-infinite chain reproduce the corresponding results obtained in [11].

[1] D. Derrida, E. Domany, and D. Mukamel, J. Stat. Phys. **69**, 667 (1992).
[2] G. Schütz and E. Domany, J. Stat. Phys. **72**, 277 (1993).
[3] B. Derrida, M. R. Evans, V. Hakim, and V. Pasquier, J. Phys. A **26**, 1493 (1993).
[4] K. Krebs and S. Sandow, J. Phys. A **30**, 3165 (1997).
[5] N. Rajewsky, A. Schadschneider, and M. Schreckenberg, J. Phys. A **29**, L305 (1996).
[6] A. Honecker and I. Peschel, J. Stat. Phys. **88**, 319 (1997).
[7] H. Hinrichsen, J. Phys. A **29**, 3659 (1996).
[8] N. Rajewsky and M. Schreckenberg, Physica A **245**, 139 (1997).
[9] N. Rajewsky, L. Santen, A. Schadschneider, and M. Schreckenberg, J. Stat. Phys. **92**, 151 (1998).
[10] M. R. Evans, N. Rajewsky, and E. R. Speer, J. Stat. Phys. **95**, 45 (1999).
[11] J. de Gier and B. Nienhuis, Phys. Rev. E **59**, 4899 (1999).
[12] J. Brankov, N. Pesheva, and N. Valkov, Phys. Rev. E **61**, 2300 (2000).
[13] B. Derrida and M. Evans, in *Nonequilibrium Statistical Mechanics in One Dimension*, edited by V. Privman (Cambridge University Press, Cambridge, 1997).
[14] B. Derrida, Phys. Rep. **301**, 65 (1998).
[15] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000).